3.3 The Power Rule

day 22

Recall $x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})$

and

$$(x+a)^n = x^n + nx^{n-1}a + \frac{n(n-1)}{2}x^{n-2}a^2 + \cdots + a^n$$

Figure out $\frac{d}{dx}(x^n)$, where n is a positive integer ([[natural number!]])

using one of the 2 limit definitions of a derivative!

$(x^n)'$ NOT PREFERRED

$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$ for $f(x) = x^n$ day 22

$= \lim_{w \to x} \frac{w^n - x^n}{w - x}$

$= \lim_{w \to x} \frac{(w-x)(w^{n-1} + w^{n-2}x + w^{n-3}x^2 + \cdots + wx^{n-2} + x^{n-1})}{(w-x)}$

$(n-1)$ x-terms + 1 w-term that has no x's

$= \lim_{w \to x} (w^{n-1} + w^{n-2}x + w^{n-3}x^2 + \cdots + wx^{n-2} + x^{n-1})$

$= x^{n-1} + x^{n-2}x + x^{n-3}x^2 + \cdots + x \cdot x^{n-2} + x^{n-1}$

$= x^{n-1} + x^{n-1} + x^{n-1} + \cdots + x^{n-1} + x^{n-1} = nx^{n-1}$

n of these

$\frac{d}{dx}(x^n) = nx^{n-1}$

NB the variable is the base

SUMMARY

NB NOT A VARIABLE nor

day 22

$\frac{d}{dx}(x^n) = nx^{n-1}$ true for every n

$\frac{d}{dx}(x^\pi) = \pi x^{\pi-1}$

$\frac{d}{dx}(x^e) = ex^{e-1}$

AND ..... a future episode . . .

$\frac{d}{dx}(e^x) = e^x$

2^x : slope of tan @ (0,1) is .693

3^x : 1.098

4^x : 1.38

e ← ? : slope of e^x at (0,1) is 1

day 22

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{when} \quad f(x) = x^n$$

$e^x = 2$

$$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} =$$

$$= \lim_{h\to 0} \frac{x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \textcircled{3}x^{n-3}h^3 + \cdots h^{n-1} - x^n}{h}$$

$$= \lim_{h\to 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + h^{n-1}}{h}$$

$$= \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \sim h^2 \sim + h^{n-2}$$

$0 \; 0 \cdots 0 \cdots 0 \to 0$

$$= nx^{n-1}$$

Conclusion $\frac{d}{dx}(x^n) = nx^{n-1}$

Sep 27-7:26 AM

day 22

$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$

$\frac{d}{dx}(x^4) = 4x^{4-1}$

$= 4x^3$

is 'x' the base? ✓

multiply by exponent

subtract one from exponent

$\frac{d}{dx}(2x^2) = 2\frac{d}{dx}(x^2) = 2(2x) = 4x$

$\frac{d}{dx}(2x^2+5x+2) = \frac{d}{dx}(2x^2)+\frac{d}{dx}(5x)+\frac{d}{dx}(2)$

$\frac{d}{dx}(5x^1) = 5 \cdot 1x^{1-1}$

$= 5x^0 = 5$

$\frac{d}{dx}(2) =$

$\frac{d}{dx}(2x^0) =$

$2(0x^{-1}) = 0$

HW/ 3.3/13-14, 19,20, 25-26, 29, 35-36, 41, 44-45

multiply out first!

simplify first!

Sep 27-8:48 AM